UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: March 2026

Module Number: CE 5204 **Module Name: Structural Analysis III (C-23)**

[Three Hours]

[Answer **all questions**, each question carries twelve marks]

Q1. a) Explain briefly the different types of yield lines?

[2 Marks]

b) Discuss briefly necessasity of yield line analysis?.

[2 Marks]

c) Isotropically reinforced hexagonal slab is simply supported along four edges while fixed supported along two edges as shown in Figure Q1. The side length of the hexagonal slab is L and overall perimeter length of the slab is , therefore, is '*6L*'. The slab carries a uniformly distributed load of intensity '*p*' per unit area.

Determine the corresponding collapse load for the yield line pattern given in Figure Q1, assuming the yield moment per unit length of both positive yield lines and negative yield lines as '*m*'.

[8 Marks]

Q2. a) Compare the charateristics of different types of shells.

[2 Marks]

b) Show that the membrane stresses in a conical shell (*with usual notations and sign convention*) are given by

$$N_\theta = P_r\, S \tan\alpha$$

$$N_s = \frac{1}{S}\int (P_r\ S \tan\alpha - P_s\ S)ds$$

Assume that the membrane stresses in a spherical shell (with usual notations and sign convention) are given by

$$\frac{N_\phi}{r_1} + \frac{N_\theta}{r_2} = P_r \qquad\qquad P_\phi r r_1 - r_1 N_\theta Cos\phi + \frac{\partial(rN\phi)}{\partial\phi} = 0$$

[4 Marks]

c) A conical shell made of thin aluminum sheets is proposed to be used as a roof of a cafeteria as shown in Figure Q2. The shell is subjected to its self-weight of "*p*" per unit surface area.

From the membrane theory derived in Part (b), determine the membrane stress resultant in the roof shell structure.

[6 Marks]

Q3. a) "Structural materials are generally far more efficient in an extensional mode rather than in a flexural mode, making the arch and shell are preferable over the beam and plate"
Do you agree with this statement? Justify your answer providing reasons.

[2 Marks]

b) Show that the membrane stresses in a spherical shell (*with usual notations and sign convention*) are given by

$$\frac{N_\phi}{r_1} + \frac{N_\theta}{r_2} = P_r \qquad\qquad P_\phi r r_1 - r_1 N_\theta Cos\phi + \frac{\partial(rN\phi)}{\partial\phi} = 0$$

[4 Marks]

c) A spherical lantern shell stiffened by the upper ring and the lower ring is shown in Figure Q3. The shell structure is having radius of R and carrying a vertical line load of 0.5 pR per unit length and a designed live load of p per unit plan area.
Determine membrane stresses caused by above loading.

[6 Marks]

Q4. a) Briefly explain the load resisting mechanism of plate elements and how it differs from that of beam elements.

[2 Marks]

b) A thin rectangular plate of dimensions $2L$ and L with thickness t is simply supported along all four edges. The plate is subjected to a transverse load given by

$$q(x,y) = q_0 \sin\left(\frac{\pi x}{2L}\right) \sin\left(\frac{3\pi y}{L}\right)$$

where q_0 is a constant.

i.) Assume a trial solution for the plate deflection. Show that the assumed trial solution satisfies the simply supported boundary conditions of the plate.

[3 Marks]

ii.) Derive the expression for the plate deflection $w(x,y)$. Hence determine the bending moments and shear forces.

[5 Marks]

iii.) Determine the maximum deflection of the plate, if $L = 1\,m$, load intensity q_0 = 200 kPa, Young's modulus $E = 200\ GPa$, Poisson's ratio $v = 0.3$, and plate thickness $t = 10\ mm$.

[2 Marks]

Governing differential equation and the expressions for bending moments and shear forces (with usual notations and sign convention) are given by,

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \qquad M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial y^2}\right) \qquad M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + v\frac{\partial^2 w}{\partial x^2}\right)$$

$$Q_x = -D\left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x\,\partial y^2}\right) \quad Q_y = -D\left(\frac{\partial^3 w}{\partial x^2\,\partial y} + \frac{\partial^3 w}{\partial y^3}\right)$$

where,

$$D = \frac{Et^3}{12(1-v^2)}$$

Q5. a) Show that, for an axisymmetrically loaded circular plate, the governing differential equation and the expressions for the radial and tangential bending moments (using the usual notation and sign conventions) are given by,

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = \frac{Q}{D} \qquad M_r = -D\left[\frac{d^2 w}{dr^2} + v\frac{1}{r}\frac{dw}{dr}\right] \qquad M_t = -D\left[\frac{1}{r}\frac{dw}{dr} + v\frac{d^2 w}{dr^2}\right]$$

Where,

$$D = \frac{Et^3}{12(1-v^2)}$$

[6 Marks]

b) A circular plate of radius *1.0 m* and thickness *20 mm* is simply supported at its edge. Assume *E = 200 GPa* and Poisson's ratio *v = 0.3.*

i.) If the maximum permissible deflection at the centre of the plate is *5 mm,* determine the maximum uniformly distributed load q that the plate can sustain.

[5 Marks]

ii.) Determine the maximum radial bending moment in the plate corresponding to the load obtained in *part (b) (i).*

[1 Mark]

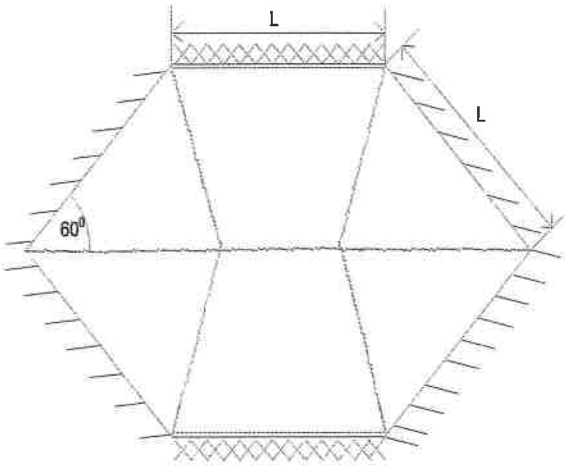

Figure Q1

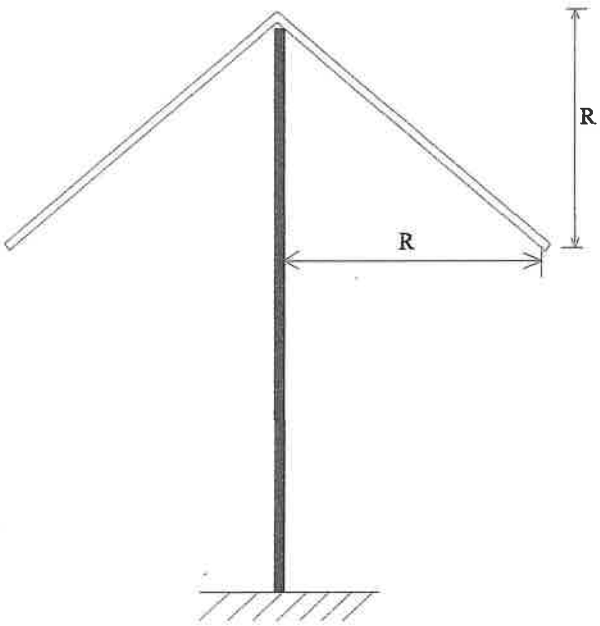

Figure Q2

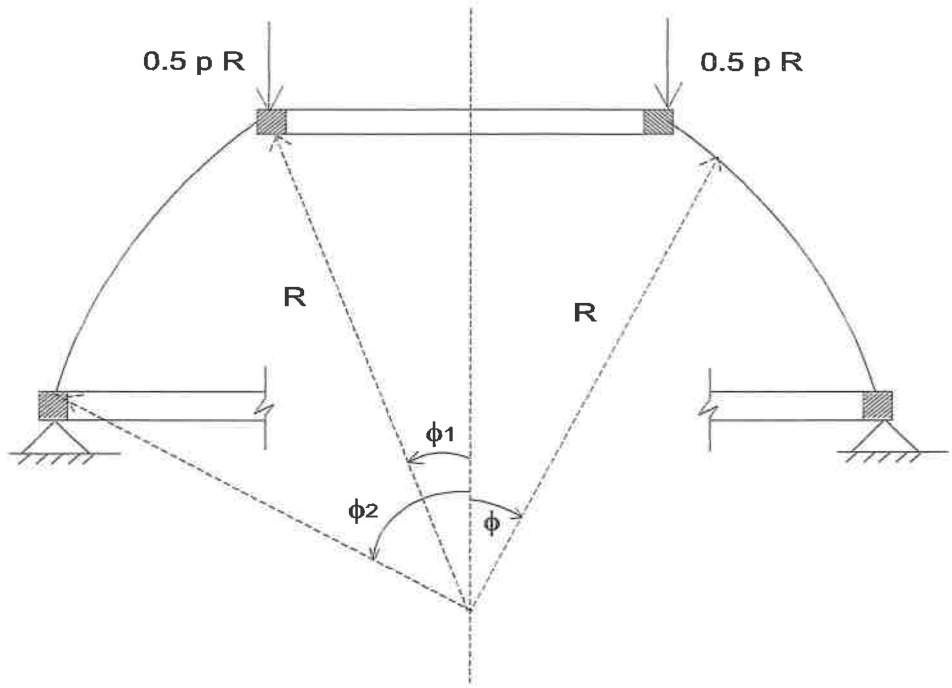

Figure Q3