



UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: March 2026

Module Number: EE5207 Module Name: Advanced Digital Communications (C-23)
[3 Hours]

[Answer all questions, each question carries ten marks]

Q1) a) Briefly explain how orthonormal basis functions are used to simplify the detection of signals in digital communication systems. [1.0 Mark]

b) A digital communication system uses two orthonormal basis functions $\Phi_1(t)$ and $\Phi_2(t)$. Three signal vectors s_1 , s_2 and s_3 in this signal space are given by

$$s_1 = (2, 0), \quad s_2 = \left(\frac{1}{\sqrt{2}}, \frac{3}{\sqrt{6}}\right), \quad s_3 = \left(-\frac{2}{\sqrt{2}}, \frac{1}{\sqrt{6}}\right)$$

and the basis functions are defined as,

$$\phi_1(t) = \begin{cases} A, & 0 \leq t < 1 \\ -A, & 1 \leq t < 2 \\ 0, & \text{otherwise.} \end{cases}$$

$$\phi_2(t) = \begin{cases} B, & 0 \leq t < 2 \\ -2B, & 2 \leq t < 3 \\ 0, & \text{otherwise.} \end{cases}$$

i) Find the values of A and B such that $\Phi_1(t)$ and $\Phi_2(t)$ are orthonormal. [1.0 Mark]

ii) Sketch the waveforms of $s_1(t)$, $s_2(t)$ and $s_3(t)$. [3.0 Marks]

c) Consider the following three signals defined over the interval $0 \leq t \leq 10$.

$$s_1(t) = 1, s_2(t) = t, s_3(t) = t^2$$

i) Use the Gram-Schmidt orthogonalization procedure to obtain an orthonormal set of basis functions $\Phi_1(t)$, $\Phi_2(t)$, $\Phi_3(t)$. [1.0 Mark]

ii) Express each of the signal $s_1(t)$, $s_2(t)$ and $s_3(t)$ as a linear combination of orthonormal basis functions. [2.0 Marks]

iii) Verify that the obtained basis functions are orthonormal by evaluating their inner products. [2.0 Marks]

Q2) a) Differentiate between the following digital modulation schemes in terms of their bandwidth efficiency, power efficiency, complexity, and error performance.

- Binary Phase Shift Keying (BPSK)
- Quadrature Phase Shift Keying (QPSK)
- M-ary Quadrature Amplitude Modulation (M-QAM)
- Frequency Shift Keying (FSK)

[4.0 Marks]

b) Choose the **most suitable digital modulation scheme** for each of the following applications and justify your choice.

- Satellite communication system with limited power but available bandwidth.
- High-speed Wi-Fi communication requiring very high data rates.
- Bluetooth communication for short-range, low-power devices.
- Deep-space communication where noise is very high.
- Mobile communication system requiring good bandwidth efficiency.

[2.0 Marks]

c) A typical M-ary PAM system uses the following amplitudes for its signal points.

$$a_k = (2k - 1 - M) A, \quad k = 1, 2, \dots, M$$

and the transmitted signals are given by,

$$s_k(t) = a_k[u(t) - u(t - T)]$$

i) Show that the average energy of the signal is

$$E_s = \frac{(M^2 - 1)}{3} A^2 T.$$

[2.0 Marks]

ii) Determine the number of bits per symbol.

[1.0 Mark]

iii) Derive an expression for the average bit energy E_b in terms of M , A and T .

[1.0 Mark]

Q3) a) "Determining the exact probability of symbol error in Phase Shift Keying (PSK) signal constellations is difficult." Do you agree with this statement? Justify your answer.

[2.0 Marks]

b) Consider M-PSK signal constellations where $M = 4$ and $M = 16$.

- i) Derive a general expression for the radius r of an M-PSK constellation in terms of minimum distance d .
[1.0 Mark]
- ii) Calculate radius for 4 - PSK and 16 - PSK.
[1.0 Mark]
- iii) Determine the energy ratio required for 16-PSK compared to 4-PSK.
[1.0 Mark]
- iv) Comment on bandwidth efficiency vs power efficiency of 4-PSK and 16-PSK.
[1.0 Mark]
- c) i) Define Gray encoding?
[1.0 Mark]
- ii) Explain why Gray coding is preferred in digital communication schemes such as QPSK or QAM.
[1.0 Mark]
- iii) Specify a Gray code for the 8 - point QAM signal constellation shown in Figure Q3(c).
[1.0 Mark]
- iv) Determine the symbol rate of the 8-point QAM signal constellation when the desired bit rate is 120 Mbps.
[1.0 Mark]

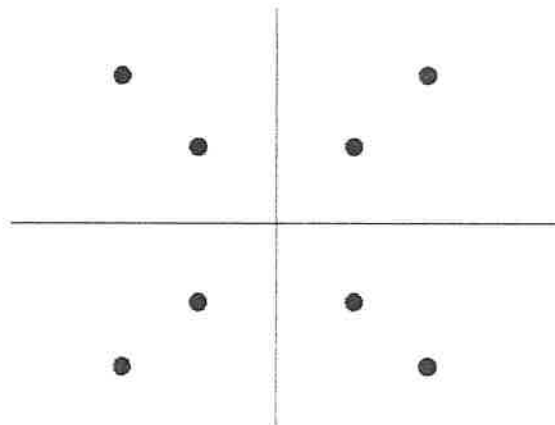


Figure Q3(c).

- Q4) a) Consider a BPSK carrier recovery system employing a squaring loop method. The received signal to the system is given by

$$r(t) = Am(t) \cos(\omega_c t + \theta),$$

where $m(t) = \pm 1$ and $\theta = \text{unknown carrier phase}$.

- i) Explain how the squaring loop carrier recovery method is used to recover the carrier frequency of the received signal.
[2.0 Marks]
- ii) Show mathematically that the recovered carrier has π - radian phase ambiguity.

[2.0 Marks]

- iii) Explain how Costas loop overcomes the π – radian phase ambiguity problem mentioned in part b)ii).

[2.0 Marks]

- b) A PAM signal is transmitted through an AWGN channel. An Early-Late synchronizer shown in Figure Q4 (b) is used for timing recovery.

- i) Explain the operation of Early-Late synchronizer shown in Figure Q4 (b). [1.0 Mark]

- ii) Derive the equation for the timing error detector. [1.0 Mark]

- iii) Explain the role of the loop filter shown in Figure Q4(b). [1.0 Mark]

- iv) Discuss advantages and limitations of Early-Late synchronizer. [1.0 Mark]

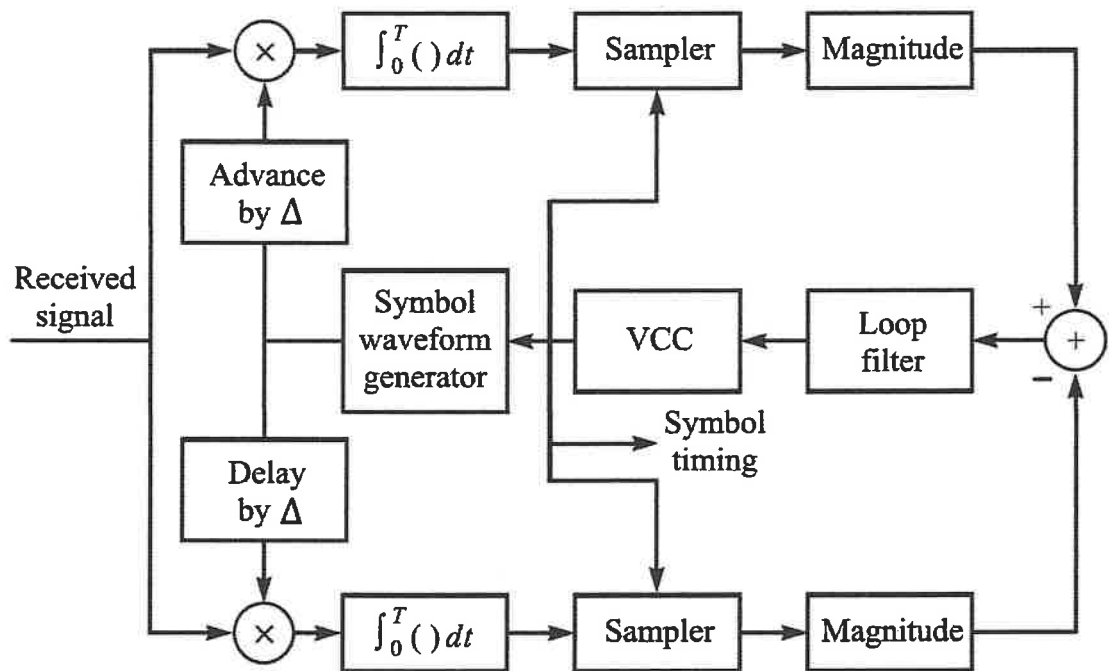


Figure Q4 (b)

- Q5) a) A digital communication system transmits one of three possible signal vectors over an AWGN channel. The signal vectors in a two-dimensional signal space are given by,

$$S_1 = (2,1), \quad S_2 = (-1,2), \quad S_3 = (-2,-2).$$

The signal received at the detector is $r = (1,1)$ and the prior probabilities of transmission are $P(S_1) = 0.5$, $P(S_2) = 0.3$, $P(S_3) = 0.2$. Assume that the noise components are independent Gaussian random variables with equal variance.

- i) Determine the transmitted signal using the Maximum A Posteriori (MAP) decision rule.

[2.0 Marks]

- ii) Determine the transmitted signal using the Maximum Likelihood (ML) decision rule. [1.0 Mark]
 - iii) Determine the transmitted signal using the Minimum Distance Detection rule. [1.0 Mark]
 - iv) Comment on whether the MAP and ML detectors produce the same decision. Justify your answer. [1.0 Mark]
- b) Consider a binary FSK communication system using orthogonal signals given by,

$$S_1 = [\sqrt{E_b}, 0]$$

$$S_2 = [0, \sqrt{E_b}]$$

The received vector is, $r = [r_1, r_2]$. Where,

$$r_1 = s_{m1} + n_1$$

$$r_2 = s_{m2} + n_2$$

Noise components n_1 and n_2 are independent Gaussian random variables with $\sigma^2 = \frac{N_0}{2}$. And m refers to the transmitted signal index. Assume $E_b = 6$ and $N_0 = 2$.

- i) Derive the decision rule based on the correlation metric. [2.0 Marks]
- ii) Derive probability of error when s_1 is transmitted. [1.0 Mark]
- iii) Derive an expression for the total probability of error. [1.0 Mark]
- iv) Calculate the numerical value of the probability of error. You may refer to the Table Q5 (b). [1.0 Mark]

Table Q 5(b)-Table of Q function Values

x	$Q(x)$	x	$Q(x)$	x	$Q(x)$	x	$Q(x)$
0	0.500000	1.8	0.035930	3.6	0.000159	5.4	3.3320×10^{-8}
0.1	0.460170	1.9	0.028717	3.7	0.000108	5.5	1.8990×10^{-8}
0.2	0.420740	2	0.022750	3.8	7.2348×10^{-5}	5.6	1.0718×10^{-8}
0.3	0.382090	2.1	0.017864	3.9	4.8096×10^{-5}	5.7	5.9904×10^{-9}
0.4	0.344580	2.2	0.013903	4	3.1671×10^{-5}	5.8	3.3157×10^{-9}
0.5	0.308540	2.3	0.010724	4.1	2.0658×10^{-5}	5.9	1.8175×10^{-9}
0.6	0.274250	2.4	0.008198	4.2	1.3346×10^{-5}	6	9.8659×10^{-10}
0.7	0.241960	2.5	0.006210	4.3	8.5399×10^{-6}	6.1	5.3034×10^{-10}
0.8	0.211860	2.6	0.004661	4.4	5.4125×10^{-6}	6.2	2.8232×10^{-10}
0.9	0.184060	2.7	0.003467	4.5	3.3977×10^{-6}	6.3	1.4882×10^{-10}
1	0.158660	2.8	0.002555	4.6	2.1125×10^{-6}	6.4	7.7689×10^{-11}
1.1	0.135670	2.9	0.001866	4.7	1.3008×10^{-6}	6.5	4.0160×10^{-11}
1.2	0.115070	3	0.001350	4.8	7.9333×10^{-7}	6.6	2.0558×10^{-11}
1.3	0.096800	3.1	0.000968	4.9	4.7918×10^{-7}	6.7	1.0421×10^{-11}
1.4	0.080757	3.2	0.000687	5	2.8665×10^{-7}	6.8	5.2309×10^{-12}
1.5	0.066807	3.3	0.000483	5.1	1.6983×10^{-7}	6.9	2.6001×10^{-12}
1.6	0.054799	3.4	0.000337	5.2	9.9644×10^{-8}	7	1.2799×10^{-12}
1.7	0.044565	3.5	0.000233	5.3	5.7901×10^{-8}	7.1	6.2378×10^{-13}