



UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: March 2026

Module Number: EE5213

Module Name: Power System Analysis (C-23)

[3 Hours]

[Answer all questions, each question carries 10 marks]

- Q1 a) i) Briefly explain the skin effect and how it affects the effective resistance of a transmission line conductor.
- ii) Why stranded conductors are used instead of solid conductors in transmission lines?
- [3 Marks]
- b) Table Q1 shows the variation of Geometric Mean Radius (GMR) with the number of strands for all Al/Cu conductors.

Table Q1

All Al or Cu Conductors	
No. of Strands	GMR
7	0.726r
19	0.758r
37	0.768r
61	0.772r
91	0.774r
127	0.776r
169	0.779r

r = radius of the conductor.

- i) What is the GMR of a single solid conductor in terms of its radius r ?
- ii) Based on Table Q1, it can be observed that the GMR approaches the value of a solid conductor as the number of strands increases. Explain why this occurs.
- iii) A stranded conductor consists of seven identical strands each having a radius r as shown in Figure Q1.b). Show that $GMR = 2.1767 r$.

Note:

$$D_{xx} = \left(\prod_{k=1}^N \prod_{m=1}^N D_{km} \right)^{\frac{1}{N^2}}$$

D_{km} is the distance between the center of strand k and center of the strand m .

When $m=k$: $D_{kk} = r' = 0.7788r$ (the self-GMR of an individual strand)

Hint: use symmetry.

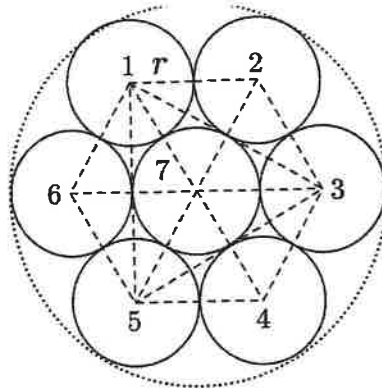


Figure Q1.b)

- iv) Using the result obtained in part iii), express the GMR of the 7-strand conductor in terms of the **overall conductor radius R** of the conductor. Hence, compare your result with the corresponding value given in Table Q1.

[4 Marks]

- c) Figure Q1.c).i) shows a 3-phase transmission line formed using twin bundled conductors.

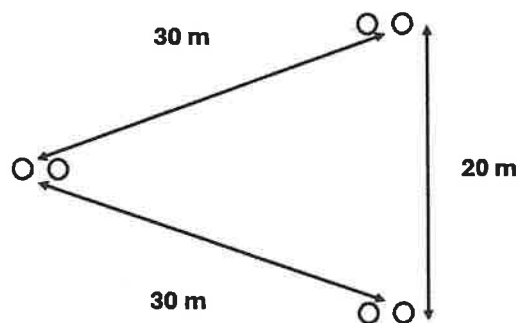


Figure Q1. c).i): Relative positions of the conductors in the transmission line (not to scale).

- Determine the per phase inductance per unit length of the 3-phase transmission line using twin bundled conductors shown in Figure Q1.c).i). The line is regularly transposed, and all the conductors are of radius **1.5 cm**. The distance between the twin conductors is **20 cm**. Other distances are as shown in Figure Q1.c).i).
- What will be the new per phase inductance per unit length if each of the twin bundles are replaced by triple bundles as shown in Figure Q1.c).ii). Each of the conductors in the bundle is of radius **1.5 cm**.

[3 Marks]

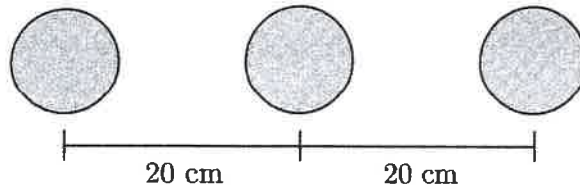


Figure Q1.c.ii): Triple bundle (not to scale).

- Q2 a) i) What is the **Ferranti effect** on transmission lines? Briefly explain the phenomenon and illustrate your answer using a phasor diagram based on the appropriate transmission line model.
- ii) Using the **long transmission line model**, derive that, for a lossless transmission line operating at **surge impedance loading (SIL)**, the voltage magnitude remains constant along the line and is equal to the receiving-end voltage.
- iii) Sketch the voltage distribution along the transmission line for the following conditions on the same graph (voltage vs distance):
- No-load condition
 - Surge impedance loading (SIL)
 - Full-load condition

Clearly label each curve and indicate the sending and receiving ends.

[4 Marks]

- b) A 345 kV, three-phase transmission line is 130 km long. The resistance per phase is $0.036 \Omega/\text{km}$ and the inductance per phase is $0.8 \text{ mH}/\text{km}$. The shunt capacitance is $0.0112 \mu\text{F}/\text{km}$. Receiving-end load is 270 MVA at 0.8 power factor lagging, with a receiving-end line voltage of 325 kV. *Frequency 60 Hz.*

Using the **medium transmission line model**, determine the following:

- i) Voltage and current at the sending end.
- ii) Percentage voltage drop at full load.
- iii) Transmission line efficiency.
- iv) The receiving-end voltage at no-load condition and the voltage regulation, if the sending-end voltage calculated in part i) is kept constant.

[6 Marks]

- Q3 a) i) Define power system stability. Briefly explain the different types of stability.
- ii) What is the swing curve? Explain the conditions under which a power system is considered stable or unstable based on rotor angle variation.
- iii) From first principles, obtain the swing equation (in per unit)

$$\frac{2H}{\omega_s} \frac{d^2 \delta}{dt^2} = P_m - P_e$$

for a synchronous generator. All the terms have their usual meanings.

[4 Marks]

- b) A 50 Hz synchronous generator having inertia constant $H = 3 \text{ MJ/MVA}$ and a direct axis transient reactance $X'_d = 0.1 \text{ pu}$ is connected to an infinite bus of 1 pu voltage through a double circuit line as shown in Figure Q3. The circuit is purely reactive and the impedances are shown on the diagram, on a common system base. The generator excitation voltage is $1.22 \angle 26.5^\circ \text{ pu}$. A solid three phase fault occurs at the middle of one of the lines (marked by point F on Figure Q3). The fault is cleared by disconnecting the faulted line.

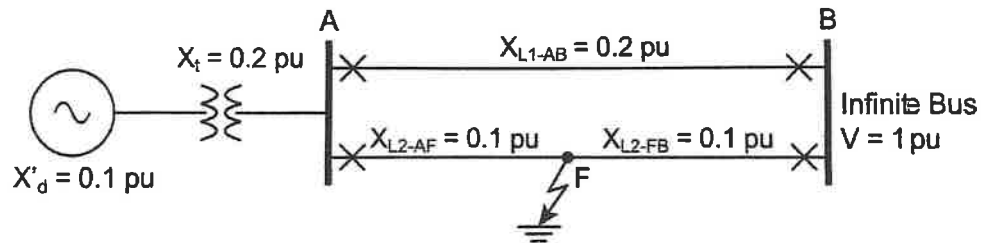


Figure Q3

- Determine the power-angle curve before the fault.
- Draw the circuit diagram to find the equivalent reactance between the generator and the infinite bus during the fault. Hence, Determine the power-angle curve during the fault.
- Determine the power-angle curve after clearing the fault.
- Determine the critical clearing angle if the generator was delivering 2 pu electrical power prior to the fault.

Hint: power-angle curve during the fault is $1.1 \sin \delta$

Note: Draw the $P - \delta$ curve and show the equality of accelerating and decelerating areas, clearly indicating the critical clearing angle and other important points.

[6 Marks]

- Q4 a) i) Define spinning reserve and briefly explain its importance in frequency control.
- ii) Explain the objectives of primary frequency control. Hence, distinguish it from secondary frequency control.

[2.5 Marks]

- b) i) Draw the **standard load frequency control (LFC) block diagram** of a single-area system with a droop-controlled governor. Clearly indicate the governor, turbine, and generator-load blocks.

Use the following notation in your diagram.

Droop constant: R

Gain of the governor: K_G

Time constant of the governor: T_G

Gain of the turbine: K_T

Time constant of the turbine: T_T

Power system time constant of generator load model: T_P

power system gain of generator load model: K_P

- ii) Assuming no change in reference power input, show that the dynamic behavior of the above system with respect to load disturbance $\Delta P_D(s)$ is given by,

$$\Delta F(s) = \Delta P_D(s) \left[\frac{-\frac{K_P}{1+sT_P}}{1 + \frac{1}{R} \left(\frac{K_T}{1+sT_T} \right) \left(\frac{K_G}{1+sT_G} \right) \left(\frac{K_P}{1+sT_P} \right)} \right]$$

in standard notation.

[2.5 Marks]

- c) Two generators A and B with rated power capacities of 200 MW and 300 MW operate in parallel serving a common load of 400 MW at 50 Hz, with generator A taking 40% share.

The governor droop of the two generators is 4% and 5% respectively. Determine the following:

- No-load and full-load frequencies of each generator.
- Load frequency and power sharing between A and B, when the total load is reduced to 300 MW.
- Maximum load that the combination can serve without subjecting any generator to overload, and the corresponding frequency. Assume that the two governors are operating in free-running mode.

[5 Marks]

- Q5 a) i) Discuss the incremental cost-based concept used for optimum dispatch of generators. **No mathematical derivations are required.**
- ii) A total demand (P_D) is to be distributed economically among two units whose fuel cost functions (F_i) in \$/hour are given as:

$$F_i = \frac{1}{2} a_i P_{Gi}^2 + b_i P_{Gi} + c_i$$

where, P_{Gi} is the active power output of i^{th} unit and a_i, b_i, c_i are constants.

When the two units are operating under economic dispatch neglecting the losses, the incremental fuel cost (λ) of the combined units can be given as:

$$\lambda = a_T P_{GT} + b_T$$

where a_T and b_T are constants, $P_{GT} = P_{G1} + P_{G2}$.

Derive analytical expressions for a_T and b_T .

[3 Marks]

- b) The fuel-cost curves for two generators are given as follows:

$$C_1 = 600 + 15P_1 + 0.05P_1^2$$

$$C_2 = 800 + 20P_2 + 0.04P_2^2$$

Neglecting network losses, calculate the optimal dispatch of generators for a total load of 1000 MW. Assume generator loading limits as follows:

$$200 \text{ MW} < P_1 < 700 \text{ MW}$$

$$100 \text{ MW} < P_2 < 500 \text{ MW}$$

[3 Marks]

- c) For the system given in Part b), the network loss (P_L) can be approximated as follows:

$$P_L = 2 \times 10^{-4}P_1^2 + 3 \times 10^{-4}P_2^2 - 4 \times 10^{-4}P_1P_2 \text{ MW}$$

If the system is operating with a marginal cost (λ) of 60 \$/hr, determine the following:

- i) Output of each generating unit.
- ii) The penalty factor of each generating unit at this operating point.
- iii) Total network losses.
- iv) Total load demand.
- v) Total operating cost.

[4 Marks]