UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Computer Engineering: May 2026

Module Number: IS5306 **Module Name: Numerical Methods(C-18)**

[Three hours]

[Answer **all questions**, each question carries twelve marks]

Q1 a) A high-frequency signal processing unit uses the characteristic equation:

$$f(x) = e^x + 4x - 10$$

to determine phase-lock thresholds, where the initial interval is [1.0, 2.0].

i.) Show that the equation $f(x) = 0$ has at least one root in the given interval using the **Intermediate Value Theorem**.

ii.) Determine the **minimum number of iterations** required for the Bisection Method to approximate the root within a tolerance of 10^{-5}.

iii.) Perform **four iterations** of the Bisection Method.
Carry out all calculations **correct to four decimal places** and present the results clearly.

[4 Marks]

b) For a system modeling signal decay, consider the function

$$f(x) = x^2 - B,$$

where B is a constant.

i.) Using the Newton–Raphson method, derive the iterative formula for solving $f(x) = 0$. Hence show that the iteration reduces to the **Babylonian method for square roots**, given by

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{B}{x_n}\right).$$

ii.) Given that $f(2) = 0$ for $f(x) = x^2 - B$, first find the value of B. Using this value, apply the Newton–Raphson method to

$$f(x) = x^3 - 2x - B$$

with initial approximation $x_0 = 2.1$, and compute x_1 and x_2 correct to five decimal places.

[4 Marks]

c) A developer proposes fixed-point iteration to solve

$$f(x) = e^x + 4x - 10 = 0$$

by rewriting it as $x = g(x)$. Two possible iteration forms are:

$$g_1(x) = \frac{10 - e^x}{4},$$

$$g_2(x) = \ln(10 - 4x)$$

i.) State the **Fixed-Point Convergence Theorem condition** for the iteration

$x_{n+1} = g(x_n)$.

ii.) Using the root interval [1.0, 2.0] , identified in Part a), evaluate the derivative of both $g_1(x)$ and $g_2(x)$. Determine which of these two forms is **mathematically guaranteed** to converge to the root.

Hint: Show your calculations for $|\, g'(x) \,|$ at the boundaries of the interval.

[4 Marks]

Q2 a) The Lagrange interpolating polynomial is defined as:

$$P_n(x) = \sum_{i=0}^{n} L_i(x)\, f(x_i)$$

i.) Define the mathematical expression for the **Lagrange Basis Polynomial,** $L_i(x)$, for a set of points $x_0, x_1, \ldots, x_n$.

ii.) Analytically prove the following key properties of the Lagrange basis polynomial $L_i(x)$:

- $L_i(x_i) = 1$.
- $L_i(x_j) = 0$ for $j \neq i$.

[3 Marks]

b) A machine learning benchmark records the latency (y) of a server based on the number of concurrent threads (x) asgiven below.

Threads (x)	1.0	3.0	4.0
Latency (y)	2.0	1.5	1.2

i.) Construct the **Newton's Divided Difference Table** for the given data points.

ii.) Use the table to derive the second-degree interpolating polynomial $P_2(x)$ and estimate the latency at $x = 2.0$. Round the final estimate to four decimal places.

iii.) If an additional data point ($x = 5.0, y = 1.0$) is added, explain why Newton's method is computationally superior to the Lagrange method in terms of "incremental" updates.

[4 Marks]

c) The power distribution across three processor cores is modeled by the system:

$$9x_1 + x_2 + x_3 = 11$$
$$2x_1 + 10x_2 + 3x_3 = 15$$
$$3x_1 + 4x_2 + 12x_3 = 19$$

i.) Show that the system is **diagonally dominant**.

ii.) Rearrange the equations into the iterative form required for the Jacobi Iterative Method.

iii.) Perform **four iterations** of the Jacobi Iterative Method using the initial guess

$$X^{(0)} = [0,0,0]^T$$

Carry out all calculations to **four decimal places**.

iv.) Contrast how the **Jacobi Method** and the **Gauss-Seidel Method** handle the updating of variables during a single iteration. Explain why the Gauss-Seidel method is generally considered more "memory-efficient" and faster to converge in most practical computer engineering applications.

[5 Marks]

Q3 a) A cooling system for a processor is tested to observe heat dissipation. The rate of heat dissipation, $Q(t)$ (in Watts), is modeled by:

$$Q(t) = 50 - 2t^{1.5}$$

i.) Approximate the total thermal energy removed, $E = \int_0^4 Q(t)\,dt$, over the first 4 seconds using the **Trapezoidal Rule** with $\boldsymbol{n = 4}$ subintervals. Round the final estimate to four decimal places.

ii.) Given that the exact value of the integral is **174.40 Joules**, compute the **absolute relative error**.

[5 Marks]

b) In a communication system, the voltage signal is given by:

$$V(t) = 4e^{-t^2}$$

i.) Use the **two-point Gauss Quadrature method** to approximate:

$$I = \int_0^1 4\,e^{-t^2}\,dt$$

Round the final estimate to four decimal places.

ii.) Calculate the absolute relative error given the exact value is **2.9886.**

Refer to the following table for the weighting factors and function argument values used in the Gauss quadrature rule.

Point	Weight Factors	Function Arguments
2	$c_1 = 1.0000$	$x_1 = -0.5773$
	$c_2 = 1.0000$	$x_2 = 0.5773$
3	$c_1 = 0.5555$	$x_1 = -0.7746$
	$c_2 = 0.8888$	$x_2 = 0.0000$
	$c_3 = 0.5555$	$x_3 = 0.7746$

[5 Marks]

c) A sensor records the discharge voltage of a backup capacitor as given below.

$t\ (s)$	0.05	0.10	0.15	0.20	0.25
$V\ (v)$	12	10.82	9.75	8.81	7.96

i.) Using the **3-point Central Difference formula with** $h = 0.05$, estimate the rate of change:

$$\frac{dV}{dt} \text{ at } t = 0.15s$$

ii.) If you were required to calculate the derivative at the **boundary** $(t = 0.05s)$, explain why the 3-point central difference formula cannot be used. State the alternative 3-point formula that would be required for such a calculation at a boundary.

[2 Marks]

Q4 a) Briefly discuss key differences between Ordinary Differential Equations (ODE) and Partial Differential Equations (PDE).

[2 Marks]

b) A cooling object follows Newton's Low of Cooling:

$$\frac{dT}{Dt} = -k(T_t - T_{amb})$$

Where,

T_t - Temperature of the object. (°C)

T_{amb} - Ambient temperature (25°C)

k - Cooling constant ($0.1\ min^{-1}$)

T_0 - Initial temperature (100°C)

Find temperature of the object after 1 minute using **modified Euler method** correct to four decimal places, with step size $h = 1$ minute.

[6 Marks]

c) Use Runge-Kutta 4th order method to find temperature of the above Part b) object after 1 minute, then compare these values with the exact temperature value.

[4 Marks]

Q5 a) Classify the following equations as linear or non-linear, and state their order.

i.) $\frac{\partial^2 U}{\partial t \partial x} = -\frac{\partial^2 U}{\partial x^2}$

ii.) $\frac{\partial T}{\partial t} + \frac{\partial^3 T}{\partial t^3} - 5T\frac{\partial T}{\partial t} = 0$

iii.) $\frac{\partial V}{\partial x} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 V}{\partial s^2} + rs\frac{\partial V}{\partial s} - rV = 0$

[3 Marks]

b) Classify the following partial differential equations as elliptic, parabolic or hyperbolic.

i.) $\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0$

ii.) $c^2 \frac{\partial^2 U}{\partial t^2} = \frac{\partial^2 U}{\partial x^2}\,; c > 0$

iii.) $8\frac{\partial^2 U}{\partial x^2} - 2\frac{\partial^2 U}{\partial x \partial y} - 3\frac{\partial^2 U}{\partial y^2} = 0$

[3 Marks]

c) Use finite divide difference method to solve the partial differential equation,

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} \text{ for } 0 \leq x \leq 1 \text{ and } 0 \leq t \leq 1$$

with the initial condition,

$$T(x, 0) = 30x \quad \text{for } 0 \leq x \leq 1$$

and the boundary conditions,

$$T(0, t) = 0$$

$$T(1, t) = 30$$

Use, $h = \frac{1}{3}$ and $k = \frac{1}{2}$, where h and k are step sizes along x and t respectively.

[6 Marks]