UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Engineering, May 2026

Module Number: EC6207 **Module Name: Digital Signal Processing (C-23)**

[Three Hours]

[Answer **all five questions**, each question carries 10 marks]

Q1 a) i) Briefly describe what are Poles and Zeros in z-domain in signal processing.

[1 mark]

ii) Briefly describe the Region of convergence (ROC) and its relationship to causality in z-domain with relevant diagrams.

[1 mark]

iii) A discrete LTI system is described by the difference equation $y(n) = 0.2y(n-1) + x(n)$, find the transfer function of the system in z-domain, $H(z)$.

[1 mark]

iv) Draw the zero-pole map and the indicate the ROC of the above Q1.a.iii system $H(z)$

[0.5 marks]

v) Find the inverse transform, $g[n]$ of the causal system $G(z)$ given below using partial fraction method.

$$G(z) = \frac{z}{(3z^2 - 4z + 1)}$$

[1.5 marks]

b) A causal, Linear Time-Invariant (LTI) discrete-time system is represented by the following second-order difference equation:

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = x[n] - \frac{1}{2}x[n-1]$$

Given that the system starts from initial rest (zero initial conditions) and is excited by an input signal $x[n] = (1/3)^n u[n]$: solve the difference equation directly in the time domain to find the total output response $y[n]$ for $n \geq 0$. You must explicitly calculate the following:

i) The homogeneous solution (natural response) $y_h[n]$ with undetermined coefficients. (Hint: assume the solution is in the form $y_h[n] = \lambda^n$)

[2 marks]

ii) The particular solution (forced response) $y_p[n]$. (Hint: Assume $y_p[n] = K(1/3)^n$)

[1.5 marks]

iii) The final total response sequence $y[n]$ by evaluating the boundary conditions at $n = 0$ and $n = 1$.

[1.5 marks]

Q2 a) A finite-duration discrete-time sequence is given by $x[n] = \{2, 1, 0, 1\}$ for $n = 0, 1, 2, 3$.

i) Compute the 4-point Discrete Fourier Transform (DFT), $X[k]$.

[1.5 marks]

ii) Determine the analytical expression for the Discrete-Time Fourier Transform (DTFT) $X(e^{j\omega})$, of the same sequence.

[1.5 marks]

iii) Prove mathematically that the 4-point DFT coefficients $X[k]$ are exact, uniformly spaced samples of the DTFT $X(e^{j\omega})$ evaluated at specific digital frequencies ω_k. State the values of ω_k for $k = 0, 1, 2, 3$.

[2 marks]

b) The first five coefficients of an 8-point DFT $X[k]$ belonging to a real-valued sequence $x[n]$ are known:

$$X[0] = 12,\ X[1] = 2 + j3,\ X[2] = 1 - j4,\ X[3] = 0,\ X[4] = -4$$

i) Using the complex conjugate symmetry properties of the DFT for real-valued sequences, determine the exact values of the remaining three coefficients: $X[5]$, $X[6]$, and $X[7]$.

[1.5 marks]

ii) State Parseval's Relation for the DFT

[1 mark]

iii) Use Parseval's relation alongside your calculated frequency-domain coefficients to determine the exact total energy of the time-domain sequence.

[1 mark]

c) Explain aliasing and Nyquist rate related to sampling of an analog signal. Then describe methods to mitigate aliasing. (Use frequency domain figures to show the aliasing and mitigation methods)

[1.5 marks]

Q3 a) i) Write down the four types of filter responses and draw the response curve of those filters. Clearly mark the passband and the stopband in the figure.

[2 marks]

ii) Write down two methods used to design a Digital filter using an Analog filter

[1 mark]

iii) Evaluate the differences between FIR and IIR filters, contrasting with at least four arguments.

[2 marks]

b) i) Convert the analog filter response, $H(s) = 1/(S^2 + 5S + 6)$ to digital filter response, $H(z)$ using IIT method, with $T = 0.1$ sec

[2.5 marks]

ii) Given the first-order high pass analog filter $G(s) = 1/(s + 10)$, use the BLT method to find an IIR digital filter $G(z)$ to approximate it for a sample period T = 0.05 s.

[2.5 marks]

Q4 a) An Infinite Impulse Response (IIR) digital system is specified by the system transfer function:

$$H(z) = \frac{2 + 3z^{-1} + z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

i) Draw the complete block diagram realization of this system using the Direct Form I structure.

[1.5 marks]

ii) Draw the block diagram using the Direct Form II (Canonic) structure.

[1.5 marks]

iii) Perform a mathematical decomposition of $H(z)$ into a series connection of first-order system sections and sketch its final Cascade realization block diagram.

[2 marks]

b) Design an FIR low-pass filter using the windowing technique to approximate an ideal brick-wall frequency response with a cutoff frequency of $\omega_c = {}^{\pi}/_{3}$.

i) Determine the analytical expression for the ideal, infinite-duration impulse response $h_d(n)$ centered at $n = 0$.

[1 mark]

ii) Truncate and shift this response using a causal Hamming Window of length $M = 5$ defined as:

$$w(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{M-1}\right), \quad 0 \le n \le M-1$$

Compute the exact numeric values of the window coefficients $w(n)$ for $n = 0, 1, 2, 3, 4$.

[2 marks]

iii) Compute the final causal filter coefficients w(n) for $n = 0, 1, 2, 3, 4$.

[2 marks]

Q5 a) Given an 8-point sequence $x[n] = \{1, 2, 1, 2, 0, 0, 0, 0\}$:

i) Tabulate the decimal-to-binary bit-reversal mapping to determine the exact reordered input sequence required for a DIT FFT algorithm layout.

[1 mark]

ii) Draw the complete and comprehensively labelled 3-stage Radix-2 DIT FFT butterfly flow graph.

[2 marks]

iii) Trace the values meticulously through each stage of your diagram to compute the final numeric 8-point DFT array $X(k)$. Ensure all twiddle factors (W_8^k) are written in their correct functional positions.

[2 marks]

b) i) State and explain three main differences between the Decimation-in-Time (DIT) and Decimation-in-Frequency (DIF) algorithms in the Fast Fourier Transform (FFT).

[1 mark]

ii) Explain the choice of selecting the number of samples N for FFT as a power of two ($N = 2^m$, m is an integer) and the usage of zero-padding in FFT.

[1 mark]

c) An aerospace telemetric unit monitors the structural integrity of a commercial aircraft's jet engine turbine. The high-speed turbine shaft is outfitted with an accelerometer sampled at a frequency of $F_s = 2000$ Hz. A diagnostic engineer applies a 2048-point DFT to the sampled vibration data stream to detect rotor blade imbalance, which is known to produce a distinct anomalous stress frequency component at 350 Hz.

i) What is the frequency resolution (spacing between bins) of this DFT in Hz?

[1 mark]

ii) In which specific DFT bin index, k should the telemetry engineer monitor to detect the 350 Hz imbalance peak? If the calculated value falls between bins, round it to the closest integer bin index k.

[1 mark]

iii) If engineering safety protocols dictate that the frequency resolution must be tightened to at least 0.4 Hz per bin, calculate the minimum number of continuous samples N that must be collected from the sensor.

[1 mark]

END OF QUESTIONS

Relevant Tables and Properties

DTFS

The DTFS can be expressed in following equations for a N periodic discrete-time signal

Synthesis Equation: $x[n] = \sum_{k=0}^{N-1} c_k \; e^{j\,2\pi\,k\,n/N}$

Analysis Equation: $c_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n] \; e^{-j\,2\pi\,k\,n/N}$

DTFT

The DTFT can be expressed in following equations

Synthesis Equation: $x[n] = \frac{1}{2\,\pi}\int_{2\pi} X(e^{j\hat{\omega}})e^{j\hat{\omega}n}d\hat{\omega}$

Analysis Equation: $X(e^{j\,\hat{\omega}}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\hat{\omega}n}$

Table 1: Basic discrete-time Fourier transform pairs.

Table of DTFT pairs	
Time-domain: $x[n]$	***Frequency-Domain:*** $X(e^{j\,\hat{\omega}})$
$\delta[n]$	1
$\delta[n - n_d]$	$e^{-j\,\hat{\omega}\,n_d}$
$r_L[n] = u[n] - u[n-L]$	$\dfrac{\sin\left(\frac{1}{2}L\,\hat{\omega}\right)}{\sin\left(\frac{1}{2}\hat{\omega}\right)}e^{-j\,\hat{\omega}\,(L-1)/2}$
$r_L[n]\,e^{j\,\widehat{\omega_0}\,n}$	$\dfrac{\sin\left(\frac{1}{2}L\,(\hat{\omega} - \widehat{\omega_0})\right)}{\sin\left(\frac{1}{2}(\hat{\omega} - \widehat{\omega_0})\right)}e^{-j(\,\hat{\omega}\,-\widehat{\omega_0})(L-1)/2}$
$\dfrac{\sin(\widehat{\omega_b}\,n)}{\pi\,n}$	$\begin{cases} 1 & \lvert\hat{\omega}\rvert \le \widehat{\omega_b} \\ 0 & \widehat{\omega_b} \le \lvert\hat{\omega}\rvert \le \pi \end{cases}$
$a^n u[n] \;\; (\lvert a\rvert < 1)$	$\dfrac{1}{1 - ae^{-j\hat{\omega}}}$
$-a^n u[-n-1] \;\; (\lvert a\rvert > 1)$	$\dfrac{1}{1 - ae^{-j\hat{\omega}}}$

Table 2: Basic discrete-time Fourier Transform properties

Table of DTFT Properties		
Property Name	***Time-Domain:*** $x[n]$	***Frequency-Domain:*** $X\left(e^{j\hat{\omega}}\right)$
Periodic in $\hat{\omega}$		$X\left(e^{j(\hat{\omega}+2\pi)}\right) = X\left(e^{j\hat{\omega}}\right)$
Linearity	$ax_1[n] + bx_2[n]$	$aX_1\left(e^{j\hat{\omega}}\right) + bX_2\left(e^{j\hat{\omega}}\right)$
Conjugate Symmetry	$x[n]\ is\ real$	$X\left(e^{-j\hat{\omega}}\right) = X^*\left(e^{j\hat{\omega}}\right)$
Conjugation	$x^*[n]$	$X^*\left(e^{j\hat{\omega}}\right)$
Time-Reversal	$x[-n]$	$X\left(e^{-j\hat{\omega}}\right)$
Delay	$x[n-n_d]$	$e^{-j\hat{\omega}n_d}X\left(e^{j\hat{\omega}}\right)$
Frequency Shift	$x[n]e^{j\hat{\omega_0}n}$	$X\left(e^{j(\hat{\omega}-\hat{\omega_0})}\right)$
Modulation	$x[n]\cos(\hat{\omega_0}n)$	$\frac{1}{2}X\left(e^{j(\hat{\omega}-\hat{\omega_0})}\right)+\frac{1}{2}X\left(e^{j(\hat{\omega}+\hat{\omega_0})}\right)$
Convolution	$x[n]*h[n]$	$X\left(e^{j\hat{\omega}}\right)H\left(e^{j\hat{\omega}}\right)$
Autocorrelation	$x[-n]*x[n]$	$\left\|X\left(e^{j\hat{\omega}}\right)\right\|^2$
Parseval's Theorem	$\sum_{n=-\infty}^{\infty}\|x[n]\|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left\|X\left(e^{j\hat{\omega}}\right)\right\|^2 d\hat{\omega}$	

DFT

The DFT can be expressed in following equations for a N length discrete time sequence.

Synthesis Equation: $x[n] = \frac{1}{N}\sum_{k=0}^{N-1}X[k]\ e^{j(2\pi/N)kn}$ for $n = 0,\ 1,\ \ldots,\ N-1$

Analysis Equation: $X[k] = \sum_{n=0}^{N-1}x[n]\ e^{-j(2\pi/N)kn}$ for $k = 0,\ 1,\ \ldots,\ N-1$

Z-transform

Table 3: Summary of important z-transform properties and pairs

Short table of z-transform pairs and properties		
$\boldsymbol{x[n]}$	$\boldsymbol{X(z)}$	**ROC**
$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	
$x[n - n_0]$	$z^{-n_0}X(z)$	
$y[n] = x[n] * h[n]$	$Y(z) = X(z)\,H(z)$	
$\delta[n]$	1	All z
$\delta[n - n_0]$	z^{-n_0}	All z except 0 (if $n_0 > 0$) or ∞ (if $n_0 < 0$)
$u[n]$	$\frac{1}{1 - z^{-1}}$	$\lvert z\rvert > 1$
$-u[-n - 1]$	$\frac{1}{1 - z^{-1}}$	$\lvert z\rvert < 1$
$a^n u[n]$	$\frac{1}{1 - az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n - 1]$	$\frac{1}{1 - az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n - 1]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$