Faculty of Technology - University of Ruhuna

Bachelor of Engineering Technology Honours

Level 2 (Semester I) End Examination – June / July 2025

Academic Year 2024/2025

Course unit: TMS2112 Linear Algebra **Duration: 2 hours**

INSTRUCTIONS TO THE CANDIDATES:

- This paper contains 4 QUESTIONS on 3 PAGES.
- ANSWER ALL QUESTIONS.
- This examination accounts for 60% of the module assignment. The total maximum mark attainable is 100. The marks assigned for each question sections are indicated in square brackets.
- Each question carries equal marks.
- All symbols carry their usual meaning.
- This is a closed book examination.
- If you have a doubt as to the interpretation of the wording of a question, make sure you take your own decision, but clearly state it on the script.
- All examinations are conducted under the rules and regulations of the University.

Question 01

(a) Let V be the collection of all real ordered pairs in which the second number is twice the first one; that is, $V = \{(x, y) \mid y = 2x, x \in \mathbb{R}\}$. If the addition and multiplication are defined, respectively, to be:

$$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2) \text{ and } \lambda \cdot (x, y) = (\lambda x, \lambda y)$$

show that V is a vector space over R with respect to the operations.

[10 Marks]

(b) Consider the set W in $\mathbb{R}^2$ defined by:

$$W = \{(x, y) \mid x - y = 0, where\ x, y \in \mathbb{R}\}$$

Use the Subspace Theorem to determine whether W is a subspace of $\mathbb{R}^2$. Clearly state your conclusion.

[07 Marks]

(c) Determine whether the set of vectors $S = \{(1, 1), (1, -1)\}$ forms a basis for $\mathbb{R}^2$. Show all necessary calculations.

[08 Marks]

Question 02

(a) The set $S = \{e^{3t}, te^{3t}, t^2e^{3t}\}$ is a basis of a vector space V of functions $f: \mathbb{R} \to \mathbb{R}$. Let D be the differential operator on V; that is, $D(f) = df/dt$. Find the matrix representation of D relative to the basis S.

[08 Marks]

(b) State the **Rank-Nullity Theorem** for a linear transformation $T: V \to W$, where V is a finite-dimensional vector space.

[03 Marks]

(c) Consider the linear transformation $T: \mathbb{P}_2 \to \mathbb{P}_2$ defined by:

$$T(ax^2 + bx + c) = ax^2 + (b + c)x + (a + b + c)$$

Describe its kernel and range and give the dimensions of each. Using your results, verify the **Rank-Nullity Theorem** for this transformation.

[14 Marks]

Question 03

Let A be the matrix:

$$A = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$$

(a) Find all eigenvalues and their corresponding eigenvectors.

[07 Marks]

(b) Find a nonsingular matrix P such that $D = P^{-1}AP$ is a diagonal matrix. Also, calculate P^{-1}.

[05 Marks]

(c) Find A^4 and $f(A)$ where $f(t) = t^4 - 3t3 - 6t^2 + 7t + 3$.

[08 Marks]

(d) Find a "real cube root" of B—that is, a matrix B such that $B^3 = A$ and B has real eigenvalues.

[05 Marks]

Question 04

Consider the electrical network shown in Figure Q4. The circuit consists of four interconnected loops with independent DC voltage sources and resistors. Let I_1, I_2, I_3, and I_4 represent the clockwise mesh currents in loops 1, 2, 3, and 4 respectively.

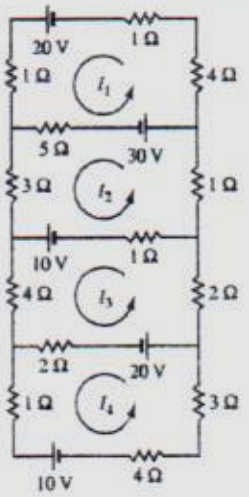


Figure Q4

(a) Apply Kirchhoff's Voltage Law (KVL) to formulate a system of four linear equations governing the mesh currents.

[08 Marks]

(b) Express this system of equations in the matrix form $AI = V$, clearly defining the resistance matrix A, the unknown current vector I, and the voltage vector V.

[05 Marks]

(c) Write down the augmented matrix for this system and solve it to find the final values of the mesh currents I_1, I_2, I_3, and I_4.

[12 Marks]

-End of the Paper-