University of Ruhuna- Faculty of Technology

Bachelor of Information and Communication Technology Honours Degree
Level 1 (Semester II) Examination, November/December 2025
Academic year 2023/2024

Course Unit: TMS1233 Discrete Mathematics **Duration: 03 hours**

INSTRUCTIONS TO CANDIDATES:

- This paper contains **6 QUESTIONS** in **06 PAGES** including this sheet.
- **ANSWER ALL QUESTIONS.** All questions carry equal marks.
- All symbols have their usual meanings.
- This is a closed book examination.
- If you have any doubt as to the interpretation of the wording of a question, make your own decision, but clearly state it on the script.
- All examinations are conducted under the rules and regulations of the University.

1)

1.1) Define the term 'proposition' and determine which of the following statements are propositions by writing "Yes" or "No".

a) Close your book.

b) Are you coming to class today?

c) Colombo is the commercial capital of Sri Lanka.

d) $10 + 5 = 50$

e) $3 + x = 12$

(20 marks)

1.2) Let p and q be the propositions

p :You drive over 65 miles per hour.
q :You get a speeding ticket.

Write the following propositions using p and q and logical connectives (including negations).

a) You do not drive over 65 miles per hour.

b) You drive over 65 miles per hour, but you do not get a speeding ticket.

c) You will get a speeding ticket if you drive over 65 miles per hour.

d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.

e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.

f) You get a speeding ticket, but you do not drive over 65 miles per hour.

g) Whenever you get a speeding ticket, you drive over 65 miles per hour.

(35 marks)

1.3) Consider the statement:
"I come to class whenever there is going to be a quiz."
Write the **converse, contrapositive**, and **inverse** of this conditional statement.

(30 marks)

1.4) Evaluate each of the following expressions :

i. $(0\ 1111 \wedge 1\ 0101) \vee 0\ 1000$

ii. $(0\ 1010 \oplus 1\ 1011) \oplus 0\ 1000$

iii. $(1\ 1011 \vee 0\ 1010) \wedge (1\ 0001 \vee 1\ 1011)$ (15 marks)

2)

2.1) Let $Q(x)$ be the statement "$x + 1 > 2x$." If the domain consists of all integers, determine truth values for the following statements:

a) $\exists x Q(x)$

b) $\forall x Q(x)$

c) $\exists x \neg Q(x)$

d) $\forall x \neg Q(x)$

(20 marks)

2.2) Translate each of the following statements into logical expressions using predicates, quantifiers, and logical connectives.

a) Something is not in the correct place.

b) All tools are in the correct place and are in excellent condition.

c) Everything is in the correct place and in excellent condition.

d) Nothing is in the correct place and is in excellent condition.

e) One of your tools is not in the correct place, but it is in excellent condition.

(25 marks)

2.3) Let $C(x)$ be the statement "x has a cat", let $D(x)$ be the statement "x has a dog", and let $P(x)$ be the statement "x has a parrot", where the domain consists of the students in your class. Express each of these the following statements in terms of $C(x)$, $D(x)$, $P(x)$, quantifiers, and logical connectives.

(a) A student in your class has a cat, a dog, and a parrot.

(b) All students in your class have a cat, a dog, or a parrot.

(c) Some students in your class have a cat and a parrot, but not a dog.

(d) No student in your class has a cat, a dog, and a parrot.

(20 marks)

2.4) Let $N(x)$ be the statement "x has visited Japan", where the domain consists of the students in your class. Express each of the following quantifications in English.

a) $\exists x N(x)$

b) $\forall x N(x)$

c) $\neg \exists x N(x)$

d) $\exists x \neg N(x)$

e) $\neg \forall x N(x)$ (35 marks)

3)

3.1) Explain the rule of inference called "Resolution".

(10 marks)

3.2) State which argument form (rule of inference) is the basis of each of the following arguments and use variables to represent it.

a) "Amali is a mathematics teacher. Therefore, Amali is either a mathematics teacher or a computer science teacher."

b) "Jerry is a mathematics teacher and a computer science teacher. Therefore, Jerry is a mathematics teacher."

c) "If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed."

d) "If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today."

e) "Kasun likes reading science books. Therefore, there exists a person who likes reading science books. "

(50 marks)

3.3) Show that the premises "If you send me an e-mail message, then I will finish writing the program," "If you do not send me an e-mail message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed."

(20 marks)

3.4) Show that the premises "Each of the 82 students in this class owns a personal computer." and "Everyone who owns a personal computer can use a word processing program." lead to the conclusion "Supun, a student in this class, can use a word processing program."

(20 marks)

4)

4.1) Find the value of the expression $(1 \cdot 1) + (\overline{(1 \cdot 0)} + 1)$ and write down translation of the equality (equality of expression and final value) into a logical equivalence.

(15 marks)

4.2) Write down the duals of the following Boolean expressions : (15 marks)

i. $(\bar{x} \cdot 0) + \overline{(y \cdot \bar{z})}$ .....

ii. $x\bar{z} + (x \cdot 1) + (\bar{x} \cdot 0)$ .....

iii. $x\,y + \overline{(x+y)}$ .....

4.3) Prove the **Absorption law**; $x(x+y) = x$ using the other identities of Boolean algebra. (20 marks)

4.4) Find the sum-of-products (SOP) expansion for the following functions using the mentioned method. (30 marks)

a) $F(x,y,z) = \overline{(x+y)} + (z \cdot y)$; using a Truth Table

b) $F(x,y,z) = (x + y) \cdot \bar{z}$; using Boolean identities.

4.5) Construct circuits from NOT gates, AND gates, and OR gates to produce the following outputs : (20 marks)

a) $x\,y + \bar{x}\bar{y}$

b) $\bar{x} \cdot \overline{(y+\bar{z})}$

c) $(x+y+z) \cdot (\bar{x}\bar{y}\,\bar{z})$

d) $y \cdot (x + x\bar{y} + \bar{z})$

5)

5.1) Define the terms of Tautologies, Contradictions and Contingencies in Propositional Equivalences. (15 marks)

5.2) Determine and classify each of the following conditional statements as Tautology, Contradiction, or Contingency. No justification is required. (40 marks)

i. $[\neg p \wedge (p \vee q)] \rightarrow q$ .....

ii. $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ .....

iii. $[p \wedge (p \rightarrow q)] \rightarrow q$ .....

iv. $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$ .....

v. $[\neg p \wedge (p \rightarrow q)] \rightarrow \neg q$ .....

vi. $[\neg q \wedge (p \rightarrow q)] \rightarrow p$ .....

vii. $(p \wedge \neg p) \rightarrow q$ .....

viii. $(p \rightarrow q) \wedge (p \rightarrow \neg q)$ .....

5.3) Show that $p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$ are logically equivalent using truth table. (15 marks)

5.4) Show that $(p \rightarrow q) \wedge (p \rightarrow r)$ and $p \rightarrow (q \wedge r)$ are logically equivalent by developing a series of logical equivalences. (Do not use a truth table to establish this equivalence).

(15 marks)

5.5) Determine whether the following compound propositions are satisfiable or not satisfiable. No justification is required.

(15 marks)

i. $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$ .....

ii. $(p \rightarrow q) \wedge (p \rightarrow \neg q) \wedge (\neg p \rightarrow q) \wedge (\neg p \rightarrow \neg q)$ .....

iii. $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$ .....

6)

6.1) Find the cardinal number of the following sets. (15 marks)

a) $A = \{x : x \in Z, -4 < x \leq 6\}$

b) $B = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, 0\}$

c) $C = \{x : x \in Z^+, x \leq 99\}$

6.2) Write down the power set of the following sets. (15 marks)

a) $A = \{0, 2, -4\}$

b) $B = \{\emptyset, 0\}$

c) $C = \{\emptyset, \{\emptyset\}\}$

6.3) State whether each of these statements is true or false. (35 marks)

a) $0 \notin \emptyset$ .....

b) $\emptyset \in \{0\}$ .....

c) $\{0\} \subset \emptyset$ .....

d) $\emptyset \subset \{0\}$ .....

e) $\{0\} \notin \{0\}$ .....

f) $\{0\} \subseteq \{0\}$ .....

g) $\{\emptyset\} \subseteq \{\emptyset\}$ .....

6.4) a) Construct a membership table to show that the Absorption law holds. (15 marks)

$$A \cup (A \cap B) = A \ , \ A \cap (A \cup B) = A$$

b) Use set builder notation and logical equivalences to establish the first De Morgan law,

$$\overline{(A \cap B)} = \bar{A} \cup \bar{B}$$

****** End of the Examination Paper******