



University of Ruhuna – Faculty of Technology
Bachelor of Engineering Technology Honors Degree
Bachelor of Information & Communication Technology Honors Degree
Level 1 (Semester II) Examination,
November/December 2025
Academic year 2023/2024

Course Unit: TMS1213: Applied Calculus II

Duration: 3 hours

- All symbols have their usual meanings.
- Answer all (05) questions.
- Calculators are not allowed.

1. The integral I is given as.

$$I = \int_1^2 \frac{1}{x^2 - x\sqrt{x^2 - 1}} dx$$

i. Show that the substitution $x = \sec \theta$ transforms I to

$$I = \int_0^{\pi/3} \frac{\tan \theta}{\sec \theta - \tan \theta} d\theta$$

(50 marks)

ii. Hence show that $I = 1 + \sqrt{3} - \frac{1}{3}\pi$.

(Hint: Use trigonometric identities)

(50 marks)

2. $f(x) = x^3 - 8x^2 + 15x$

- i. What are the x intercepts and y intercepts of the above function $f(x)$. (10 marks)
- ii. Find the intervals where the function is increasing and decreasing.
(you may use $\sqrt{19} \approx 4.35$) (30 marks)
- iii. Find the intervals where the function is concave up and concave down. (30 marks)
- iv. Find inflection points. (10 marks)
- v. Find the behavior of the function at extreme ends ($x \rightarrow \infty$; $x \rightarrow -\infty$) (10 marks)
- vi. Based on the information you have obtained from the above parts, sketch the shape of the function $f(x)$ on a 2D cartesian plane. (10 marks)

3. Calculate the Arc length of the following function over the given interval.

$$y = \frac{1}{3}(x^2 + 2)^{3/2} \text{ over the interval } [1,5]$$

(100 marks)

4.

- i. Sketch the function $f(x)$ and $g(x)$ on the same plot.

$$f(x) = x^2 + 2$$

$$g(x) = 3x^2$$

(20 marks)

- ii. Clearly show (with shading) the area that is bound by the functions $f(x)$, $g(x)$ and the lines $y = 6$ and $y = 3$ in the 1st(first) quadrant of the 2D cartesian plane. (10 marks)
- iii. Calculate the volume of the solid object formed by revolving the above-mentioned area around the y - axis. (70 marks)

5. Calculate the area of the surface obtained by rotating $y = \sqrt{x}$ about the x - axis within the interval $[4,9]$. (100 marks)

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