UNIVERSITY OF RUHUNA

FACULTY OF FISHERIES AND MARINE SCIENCES & TECHNOLOGY

Academic Year 2022/2023

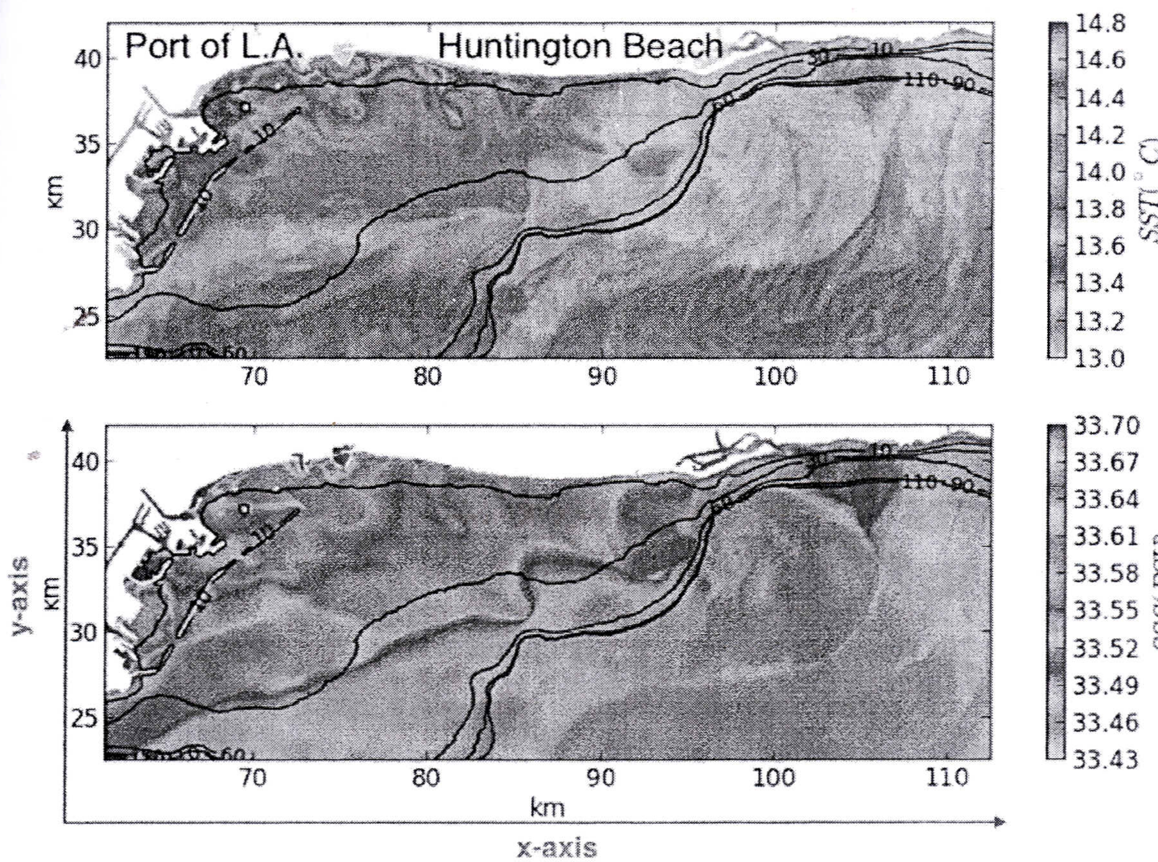
Bachelor of Science Honours in Marine and Freshwater Sciences Degree
Level III Semester I Examination – July 2024

OCG 3172: Mathematics II **Time: 02 (Two) hours**

Answer **any four (04)** questions. Each question carries equal marks.

Q1

The following figure shows the sea surface temperature and salinity of the coast of Southern California from a ROMS (Regional Oceanic Modeling System) simulation. The top panel shows sea surface temperature (SST) and the bottom panel shows sea surface salinity (SSS).



(a) What do you mean by "a scalar field"? Explain giving examples except the temperature field. [15 marks]

(b) The temperature field at the surface of the ocean is represented as a function of x by $T(x) = 3x^4 + e^{-2x} + \sin(3x + 1)$. Find the derivative of $T(x)$ and explain the meaning of it. [25 marks]

(c) Now, the temperature is written as a function of x and y
$G(x, y) = \cos^2(2x + 5) + \sin(3y + 2)$. Find

(i) the first partial derivatives of $G(x, y)$. [15 marks]

(ii) the second partial derivatives of $G(x, y)$. [20 marks]

(iii) the total differential of $G(x, y)$ at the point $\left(\frac{\pi - 15}{6}, \frac{3\pi - 4}{6}\right)$. [15 marks]

(d) Is $G(x, y)$ in part (c) homogeneous? Justify your answer. [10 marks]

Q2

(a) Let $\underline{a}$ and $\underline{b}$ be any two non-zero vectors in $\mathbb{R}^3$. Define, in usual notation,

(i) the scalar product $\underline{a} \,.\, \underline{b}$,

(ii) the vector product $\underline{a} \times \underline{b}$. [20 marks]

(b)

(i) Find the value of m, such that the points $(-1, -1, 2)$, $(2, m, 5)$ and $(3, 11, 6)$ are collinear. [15 marks]

(ii) Let $\underline{a} = 2\,\underline{i} - \underline{j} + \underline{k}$, $\underline{b} = \underline{i} + \underline{j} - 2\underline{k}$ and $\underline{c} = \underline{i} + 3\,\underline{j} - \underline{k}$. Find λ such that $\underline{a}$ is perpendicular to $\lambda\underline{b} + \underline{c}$. [15 marks]

(iii) Find all vectors of magnitude $10\sqrt{3}$ that are perpendicular to the plane containing the two vectors $\underline{i} + 2\,\underline{j} + \underline{k}$ and $-\underline{i} + 3\,\underline{j} + 4\,\underline{k}$. [20 marks]

(iv) Find parametric equations of the line passing through the point $(4, -1, 3)$ and parallel to the vector $(1, 4, -3)$. [20 marks]

(v) Find the vector equation of a plane passing through a point $(3, 4, 2)$, and is perpendicular to a line with direction cosines $(2, -3, 1)$. [10 marks]

Q3

(a) Let $\underline{a}$, $\underline{b}$ and $\underline{c}$ be any three non-zero vectors in $\mathbb{R}^3$. Define, in the usual notation,

(i) the scalar triple product,

(ii) the vector triple product. [10 marks]

(b) Find the volume of the tetrahedron having vertices

$$(-\underline{j} - \underline{k}),\ (4\underline{i} + 5\underline{j} + q\underline{k}),\ (3\underline{i} + 9\underline{j} + 4\underline{k}) \text{ and } 4(-\underline{i} + \underline{j} + \underline{k}).$$

Find the value of q for which these four points are coplanar. [30 marks]

(c) A particle moves along the curve $\underline{r} = (t^3 - 4t, t^2 + 4t, 8t^2 - 3t^3)$, where t is time.

(i) Find the velocity ($\underline{v}$) and the acceleration ($\underline{a}$) of the particle when $t = 2$. [20 marks]

(ii) Find the velocity in the direction of the tangent to the curve when $t = 2$. [10 marks]

(iii) Find the magnitude of the tangential component of its acceleration when $t = 2$. [10 marks]

(d) Verify that $\underline{i} \times (\underline{a} \times \underline{i}) + \underline{j}\,(\underline{a} \times \underline{j}) + \underline{k} \times (\underline{a} \times \underline{k}) = 2\underline{a}$, where $\underline{a} = a_1\underline{i} + a_2\underline{j} + a_3\underline{k}$. [20 marks]

Q4

(a) Use reduced row echelon form and solve the system of linear equations given by

$$x_1 + x_2 + 2x_3 = 8$$
$$-x_1 - 2x_2 + 3x_3 = 1$$
$$3x_1 - 7x_2 + 4x_3 = 10$$

[15 marks]

(b) Let $\underline{v}_1, \underline{v}_2, \underline{v}_3, \ldots, \underline{v}_n$ be given n vectors in $\mathbb{R}^n$. Explain what is meant by

(i) a vector $\underline{v}$ in $\mathbb{R}^n$ is a linear combination of $\underline{v}_1, \underline{v}_2, \underline{v}_3, \ldots, \underline{v}_n$.

(ii) $\underline{v}_1, \underline{v}_2, \underline{v}_3, \ldots, \underline{v}_n$ are linearly dependent. [20 marks]

(c) Examine the following vectors for linear dependence.

$\underline{v}_1 = (1,2,4),\ \underline{v}_2 = (2,-1,3),\ \underline{v}_3 = (0,1,2),\ \underline{v}_1 = (-3,7,2)$ [35 marks]

(d) For any real vector spaces V and W, define the linear transformation $L : V \to W$.

[10 marks]

Let $L: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $L\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{bmatrix} 2a_1 - 1 \\ 3a_2 \\ 0 \end{bmatrix}$. Determine whether L is a linear transformation or not. [20 marks]

Q5

(a) Briefly explain the following: [15 marks]

(i) Cartesian coordinate system,

(ii) Cylindrical coordinate system,

(iii) Spherical coordinate system.

(b) Express the point $\left(2, \frac{4\pi}{3}, 8\right)$ given by cylindrical coordinates in both cartesian and spherical coordinates. [20 marks]

(c) Let

$$u = x + y + z,$$
$$v = x^2 + y^2 + z^2,$$
$$w = yz + zx + xy\,.$$

In the usual notation prove that $grad\ u$, $grad\ v$ and $grad\ w$, are coplanar. [25 marks]

(d) Find the divergence and curl of $\underline{V} = xyz\,\underline{i} + 3xy^2\,\underline{j} + (xz^2 - y^2z)\,\underline{k}$ at $(2,-1,1)$.

[40 marks]

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