University of Ruhuna

Bachelor of Science General Degree Level I (Semester I) Examination

June 2023

bject: Applied Mathematics/Industrial Mathematics
ourse Unit: AMT111β/IMT111β(Classical Mechanics-I)

Time: Two (02) Hours

<u>Answer all Questions.</u>

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1. (a) The parametric equations for the trajectory of a particle of mass m moving in a force field are given by $x(t) = \pi \sin(\frac{\pi t}{3})$, $y(t) = \pi \cos(\frac{\pi t}{3})$, and $z(t) = \frac{\pi t^2}{36}$. If the direction of the force field acting on the particle is reversed at $t = 6$, find

   (i) the velocity of the particle, *[10 marks]*

   (ii) the speed of the particle. *[10 marks]*

   (iii) the acceleration of the particle, and *[10 marks]*

   (iv) force acting on the particle. *[10 marks]*

   for for $0 < t < 6$. Also calculate

   (i) the work done on the particle when it moves from $t = 0$ to $t = 3$, and *[10 marks]*

   (ii) velocity of the particle at time at $t = 9$. *[10 marks]*

   (b) A particle of unit mass is projected vertically upward with an initial speed u_0 from a point of height h above the ground level. The particle moves under the gravity and resistant force kv^2, where v is the speed, and k is a constant. Find

   (i) the maximum height H reached by the particle from the ground level, and *[20 marks]*

   (ii) the speed v_0 of the particle when it reaches the ground . *[20 marks]*

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2. a) In cylindrical polar coordinates, (r, θ, z), show that the velocity and acceleration of a moving particle are given by

   (i) $\underline{\mathbf{v}} = \dot{r}\hat{\underline{\mathbf{r}}} + r\dot{\theta}\hat{\underline{\theta}} + \dot{z}\hat{z}$ *[20 marks]*

   (ii) $\underline{\mathbf{a}} = (\ddot{r} - r\dot{\theta}^2)\hat{\underline{\mathbf{r}}} + \dfrac{1}{r}\dfrac{d}{dt}(r^2\dot{\theta})\hat{\underline{\theta}} + \ddot{z}\hat{z}.$ *[20 marks]*

   b) A particle is projected horizontally at the point $r = a$ with speed v along the smooth inner surface given in cylindrical polar coordinates by $z = \dfrac{r^2}{2a}$, whose axis is vertically upward. If the particle is at the point (r, θ, z) at time t,

   (i) show that the kinetic energy, T of the particle is given by
   $T = \dfrac{m}{2}\left(r^2\dot{\theta}^2 + \dot{r}^2 + \dot{z}^2\right),$ *[10 marks]*

(ii) obtain the energy equation, $\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2 + 2gz = C_1$ and $r^2\dot{\theta} = C_2$, where C_1 and C_2 a constants, [15 marks]

(iii) find C_1 and C_2, [10 marks]

(iv) show that $\left(\frac{a}{2z} + 1\right)\dot{z}^2 + \left(\frac{a}{2z} - 1\right)v^2 + g(2z - a) = 0$, [10 marks]

(v) derive an expression for $\ddot{z}$ at $t = 0$, and [10 marks]

(vi) show that initially the particle goes up or down according to $v^2 \gtrless ga$. [05 marks]

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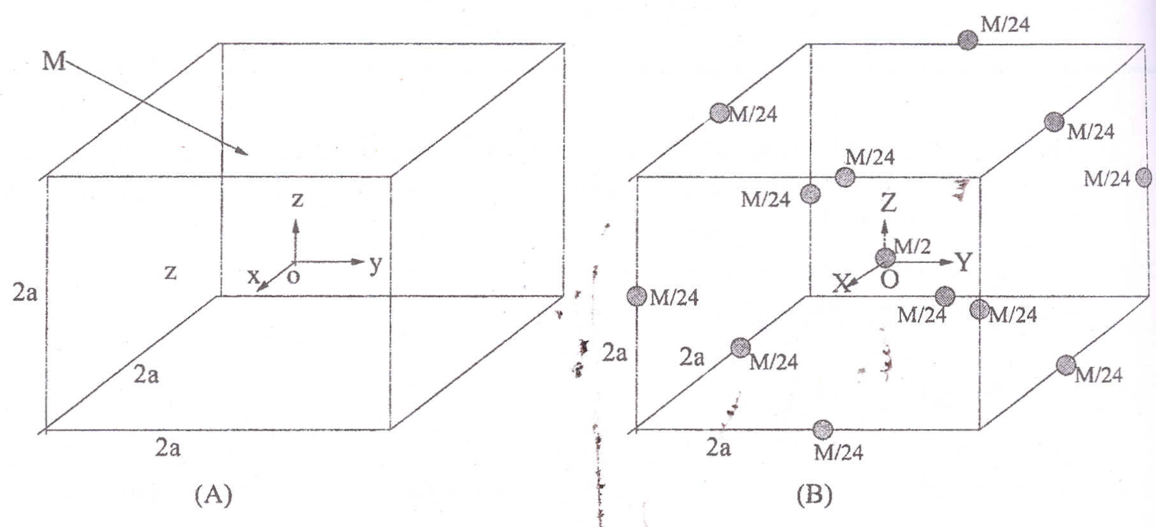
3. a) Consider the two mass systems shown in the following figure.



- **System (A):** A uniform cuboid of mass M and a side $2a$. The center of the cuboid is o and ox, oy, oz are parallel to the sides of the cuboid.
- **System (B):** A light cuboid shape frame of a side $2a$ such that 12 point masses, $M/24$, are attached at the midpoints of each edges, and point mass $M/2$ is at the center, O, where OX, OY, OZ coordinate axes are parallel to the sides of the frame.

(i) Find the moment of inertia, I_{XX}, of the mass system (B).

(ii) Find the product of inertia, I_{XY}, of the mass system (B).

(iii) Show that above two systems, (A) and (B), are equimomental. [40 marks]

[You may assume that the moment of inertia of a cuboid of mass M and side $2a$ about an axis passing through the center and parallel to a side of the cuboid is given by $2Ma^2/3$.]

b) A rigid body is free to rotate about its center of gravity, G, without external forces. The principal moments of inertia about G are $6I$, $3I$ and I units respectively. Initially the body is given an angular velocity $\underline{\omega_0} = (k, 0, 3k)$. Here I and k are constants. Show that after time t, the angular velocity $\underline{\omega} = (\omega_1, \omega_2, \omega_3)$ satisfies the equations

$$5\omega_1^2 + \omega_2^2 = 5k^2$$
$$9\omega_2^2 + 5\omega_3^2 = 45k^2$$
$$\omega_2 = -\sqrt{5}k\tanh(\sqrt{5}kt).$$

Also, find ω_1 and ω_3. [60 marks]

Continued.