UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: March 2026

Module Number: ME 5301 **Module Name: Advanced Control Systems - C23**

[**Three** Hours]

[Answer **all questions**, Q1 carries **20** marks. Other questions carry **10** marks]

Important:

Some necessary equations and a partial table of Laplace transformation pairs have been provided in the question paper. **You may make additional assumptions**, if necessary, by clearly stating them in your answers. Some standard notations may have been used without defining them explicitly.

The standard form of a second-order system is $G(s) = \frac{k\omega_n^2}{S^2+2\zeta\omega_n S+\omega_n^2}$;

$T_s = \frac{4}{\zeta\omega_n}$ ($\pm 2\%$ settling time);

Percentage Overshoot = $e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)} \times 100$;

Bilinear transform $s = \frac{2}{T}\frac{(Z-1)}{(Z+1)}$

Q1. A single variable discrete time system is described by

$$X_k = X_{k-1} + W_{k-1}$$

$$Z_k = X_k + V_k$$

where,

x_k : state (scalar)

z_k : measurement or output (scalar)

$w_k \sim \mathcal{N}(0, Q)$: process noise (i.e. w is a random variable that follows a normal (Gaussian) distribution with mean = 0 and Variance = Q)

$v_k \sim \mathcal{N}(0, R)$: measurement noise

Given:

Initial estimate: $\hat{x}_0 = 2$

Initial covariance: $P_0 = 1$

Process noise variance: $Q = 0.5$

Measurement noise variance: $R = 2$

Measurements corresponding to $k = 1$ and $k = 2$ are,

$z_1 = 3$

$z_2 = 2$

a) State the prediction and update equations for the scalar (i.e. single variable) Kalman filter.

[3.0 Marks]

b) Compute the predicted state $\hat{x}_1^-$ and predicted covariance P_1^- (i.e. predicted state and predicted covariance at step $k = 1$, **before** seeing Z_1).

[4.0 Marks]

c) Compute the Kalman gain K_1.

[2.0 Marks]

d) Compute the updated estimate $\hat{x}_1$ and updated covariance P_1. (i.e. state and covariance at step $k = 1$, **after** seeing Z_1).

[4.0 Marks]

e) Repeat steps (b), (c) and (d) for time step $k = 2$.

[5.0 Marks]

f) Comment on why the Kalman gain changed from step $k = 1$ to $k = 2$.

[2.0 Marks]

Q2. a) The dynamics of an open-loop system is given by the following transfer function. $G_P(s) = \frac{1}{2(s+4)}$. The system is driven by an amplifier having the transfer function $G_A(s) = \frac{1}{s}$.

i. Draw the block diagram of the respective closed-loop system having unity negative feedback and a single proportional controller of gain K in the forward path. Obtain the characteristic equation.

[2.0 Marks]

ii. Obtain the break-away point of the root locus and the corresponding value of the gain K.

[2.0 Marks]

iii. Find the roots of the characteristic equation when K has values 0, 4, 8, 12, and 16.

[2.0 Marks]

iv. Draw the root locus of the system indicating important points in the s-plane (Graph sheets are not necessary).

[2.0 Marks]

b) The characteristic equation of a system controlled by a single feedback gain K is shown below. Obtain the settling time of the system, when the controller gain K is selected such that both roots are equal.

$$1 + \frac{Ks(s+4)}{s^2 + 2s + 2} = 0$$

[2.0 Marks]

Q3. A first-order system has transfer function $G(s) = {}^{1}/_{(s+2)}$. The plant control signal is routed through an amplifier having transfer function $G_A(s) = {}^{1}/_{(s+4)}$.

a) Draw the block diagram of the closed-loop negative feedback system with a single feedback gain and obtain the characteristic equation.

[1.0 Mark]

b) The design requirements state that the maximum overshoot is 4% and system must settle down within 2.5 seconds according to the 2% settling time criteria. Obtain the required roots of the characteristic equation.

[2.0 Marks]

c) According to the requirements and results obtained in Q3.b), show that a proportional-only feedback gain is not sufficient to achieve the expected performance from the system.

[2.0 Marks]

d) Suggest a suitable compensator to achieve the expected performance as stated in Q3.b). Show your answer as an additional block to the Q3.a), by redrawing the block diagram with the additional block. Use variables to indicate still unknown values.

[1.0 Mark]

e) If the compensator used to achieve the expected performance as given in Q3.b), has zero located at $(-6, j0)$, find the location of the pole of the compensator.

[2.0 Marks]

f) Briefly explain how the roots of the characteristic equation influence a system's response to an input. (Maximum of 5 sentence answer is expected.)

[2.0 Marks]

Q4. a) Characteristic polynomial P of a system is given by $P(S) = S^4 + S^3 + S^2 + S + K$.

i. Obtain Routh's array of the polynomial P(S).

[1.0 Mark]

ii. Determine the values of K which will make the system **unstable.**

[2.0 Marks]

b) A system G(S) is specified as $G(S) = {}^{K}/_{(1+TS)}$.

i. Obtain the frequency response of the system in $a + jb$ form.

[2.0 Marks]

ii. Obtain the magnitude and the phase of the system.

[2.0 Marks]

iii. Fill the following table using information from the above system G(S).

ω (rad/s)	*Magnitude*	*Phase* (degrees)
0		
${}^{1}/_{T}$		
∞		

[1.0 Mark]

iv. Plot the frequency response of the system in a complex plane as a polar plot indicating important details. (No graph sheets are required.)

[2.0 Marks]

Q5. **Figure Q5** shows a rotary inverted pendulum system and the corresponding free-body diagram. The motor of the system rotates the arm, and the pendulum is pivoted to the end of the arm as shown. The pivot joint is free to rotate and has negligible friction and damping. The goal of a proposed control system is to keep the pendulum upright by controlling the arm of the system. Without active control, the pendulum will not stay upright. The system uses an armature-controlled brushed permanent magnet DC motor.

a) Derive the differential equations describing the electrical and mechanical domains of the motor and convert them to the Laplace domain assuming zero initial conditions. (Note: The DC motor has an inductance L, resistance R, Inertia J, damping constant B, supply voltage v(t), shaft speed w(t), Torque T, back emf constant k_e and torque constant k_t)

[4.0 Marks]

b) Build a non-linear system dynamics model to simulate this rotary inverted pendulum system, using a block diagram. In your answer, include details about critical configurations, locations of the sensors, joints, and forces. State the expected function of each of the blocks used. Assume that the input to the simulator is the torque and the output is the pendulum angle at this stage.

[2.0 Marks]

c) Indicating the system modeled in Q5.b) using a single block, draw a suitable closed-loop control system using a block diagram to keep the pendulum at the upright position. Clearly indicate the controller you are using with reasons.

[1.0 Mark]

d) The motor driver of DC motor has a current limit of ± 5 A and a dead band of ± 0.1 A. The DC motor accepts current as input and outputs torque. Include these details and redraw an improved model.

[1.0 Mark]

e) Suggest a suitable sensor to measure the angle of the pendulum with sufficient accuracy and precision. If you are to implement this system as a physical prototype, what is the expected accuracy?.

[2.0 Marks]

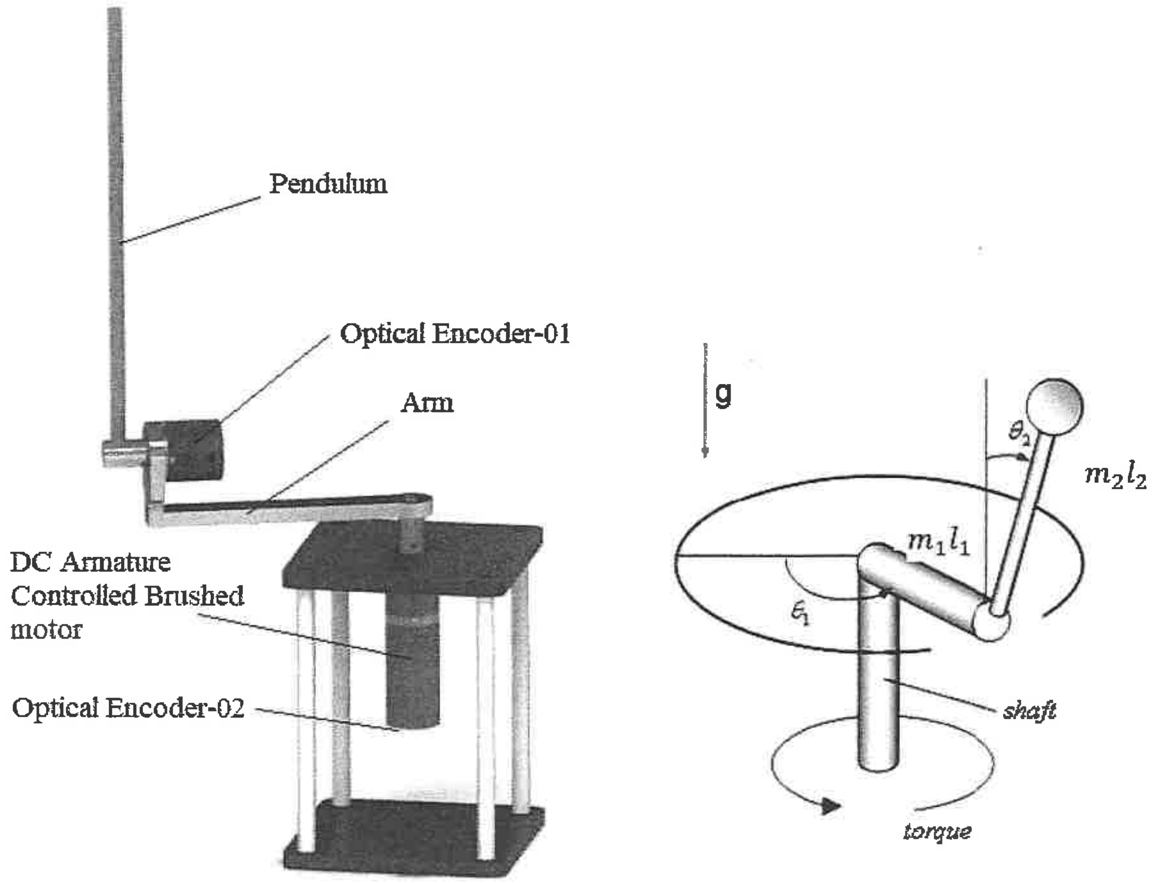
Pendulum
Optical Encoder-01
Arm
DC Armature Controlled Brushed motor
Optical Encoder-02
g
θ_2
$m_2 l_2$
$m_1 l_1$
θ_1
shaft
torque

Figure Q5

Laplace Transforms Table

$f(t) = L^{-1}\{F(s)\}$	$F(s)$	$f(t) = L^{-1}\{F(s)\}$	$F(s)$
$a \quad t \geq 0$	$\frac{a}{s} \quad s > 0$	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$at \quad t \geq 0$	$\frac{a}{s^2}$	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
e^{-at}	$\frac{1}{s+a}$	$\sin(\omega t + \theta)$	$\frac{s \sin\theta + \omega \cos\theta}{s^2 + \omega^2}$
te^{-at}	$\frac{1}{(s+a)^2}$	$\cos(\omega t + \theta)$	$\frac{s \cos\theta - \omega \sin\theta}{s^2 + \omega^2}$
$\frac{1}{2} t^2 e^{-at}$	$\frac{1}{(s+a)^3}$	$t \sin \omega t$	$\frac{2\omega s}{(s^2 + \omega^2)^2}$
$\frac{1}{(n-1)!} t^{n-1} e^{-at}$	$\frac{1}{(s+a)^n}$	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$
e^{at}	$\frac{1}{s-a} \quad s > a$	$\sinh \omega t$	$\frac{\omega}{s^2 + \omega^2} \quad s > \lvert\omega\rvert$
te^{at}	$\frac{1}{(s-a)^2}$	$\cosh \omega t$	$\frac{s}{s^2 + \omega^2} \quad s > \lvert\omega\rvert$
$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
$\frac{1}{a^2}[1 - e^{-at}(1+at)]$	$\frac{1}{s(s+a)^2}$	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
t^n	$\frac{n!}{s^{n+1}} \quad n = 1, 2, 3$	$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}} \quad s > a$	$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 + \omega^2}$
$t^n e^{-at}$	$\frac{n!}{(s-a)^{n+1}} \quad s > a$	$1 - e^{-at}$	$\frac{a}{s(s+a)}$
$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}} \quad s > 0$	$f(t - t_1)$	$e^{-t_1 s} F(s)$
$g(t) \cdot p(t)$	$G(s) \cdot P(s)$	$f_1(t) \pm f_2(t)$	$F_1(s) \pm F_2(s)$
$\int f(t)dt$	$\frac{F(s)}{s} + \frac{f^{-1}(0)}{s}$	$\delta(t) \quad \textit{Unit impulse}$	$1 \quad \textit{all } s$
$\frac{df}{dt}$	$sF(s) - f(0)$	$\frac{d^2 f}{df^2}$	$s^2 F(s) - sf(0) - f'(0)$
$\frac{d^n f}{dt^n}$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - s^{n-3} f''(0) - \cdots - f^{n-1}(0)$		